

ANGZ L → SEE RAW SCORES → LAB
8 1 1 13 grade [8, 1, 1, 13, ..., 2, 2, 2]

SEC1 SNAFU Q2
GIVE YOU Q2 RAW
SCORE

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TKRV
Exam 2
(SCORES)

Professor

STATISTICS AND PROBABILITY

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click on STT200_Sp09

GOLD STANDARD ANSWER

$\{\$1a, \$1b, \$5c\}$

Jack draws a bill first

Jill draws second

from the **two** bills then remaining



$$P(\text{Jill } \$5) = 2/6 = 1/3$$

same as Jack

CONDITIONING

TOLD JACK 1.

NEW CONDITIONAL
(REVISED) PROB
FOR JILL 5. GIVEN
INFO JACK GOT 1.

WRITE $P(\text{JILL } 5 | \text{JACK } 1)$

THINK GIVEN
JILL FACES

$\left\{ \begin{array}{l} 1 \\ 5 \end{array} \right\}$
INTUIT $P(\text{JILL } 5 | \text{JACK } 1)$
 $= \frac{1}{2}$

MULTIPLY $P(AB) = P(A)P(B|A)$

GOLD STANDARD ANSWER

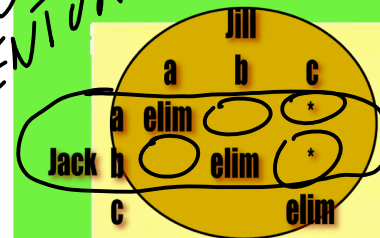
$\{\$1a, \$1b, \$5c\}$

Jack draws a bill first

Jill draws second

from the two bills then remaining

CONDITIONING
EVENT JACK 1



$P(\text{JILL } \$5)$
 $= 2/6 = 1/3$
same as Jack

DEF OF COND'L

$P(B|A) = P(AB) / P(A)$

DEFINE THIS COND'L

PROB AS REGULAR

PROB IN REDUCED.

MODEL $2/4 = \frac{1}{2}$

From
REG
PROB

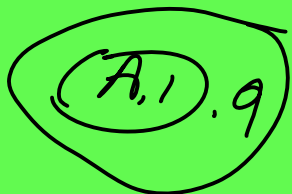
$2/6 / (4/6) = P(\text{JILL } 5) / P(\text{JACK } 1)$

EX 1. GIVEN INFO $P(A) = .1$ $P(B) = .3$

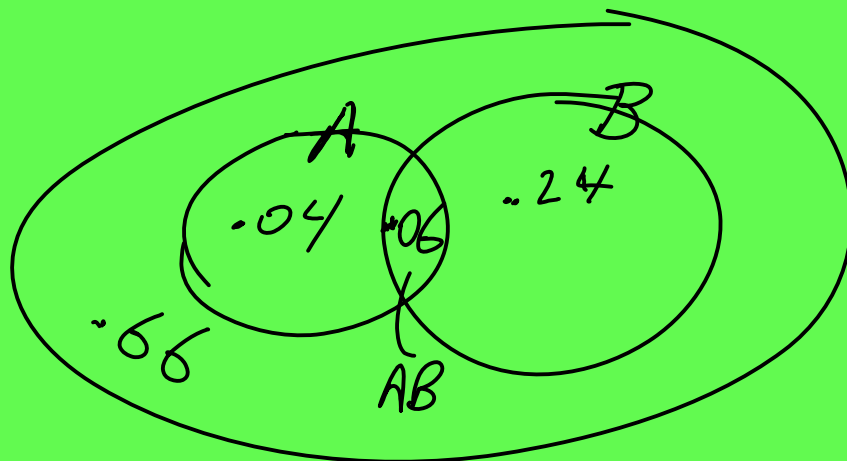
$AB \subseteq A$

$AB \subseteq B$

$P(AB) = .06$



VENN



SO $P(AB) = .06$

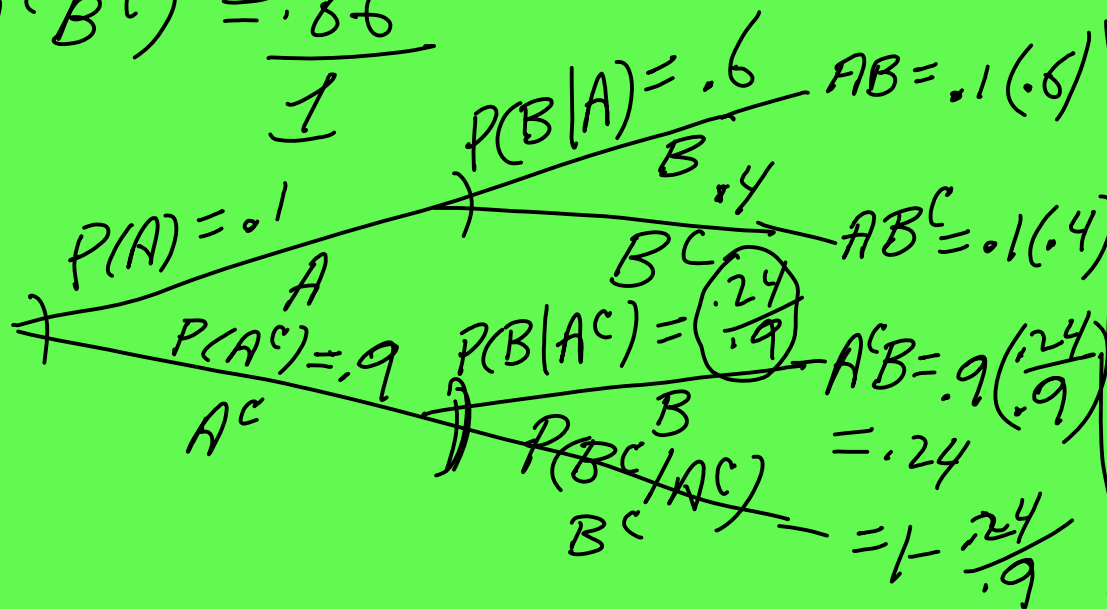
$P(AB^c) = .04$

$P(A^cB) = .24$

$P(A^cB^c) = .66$

1

TREE



DEF $P(B|A) = \frac{P(AB)}{P(A)}$
 $= .06 / .1 = .6$

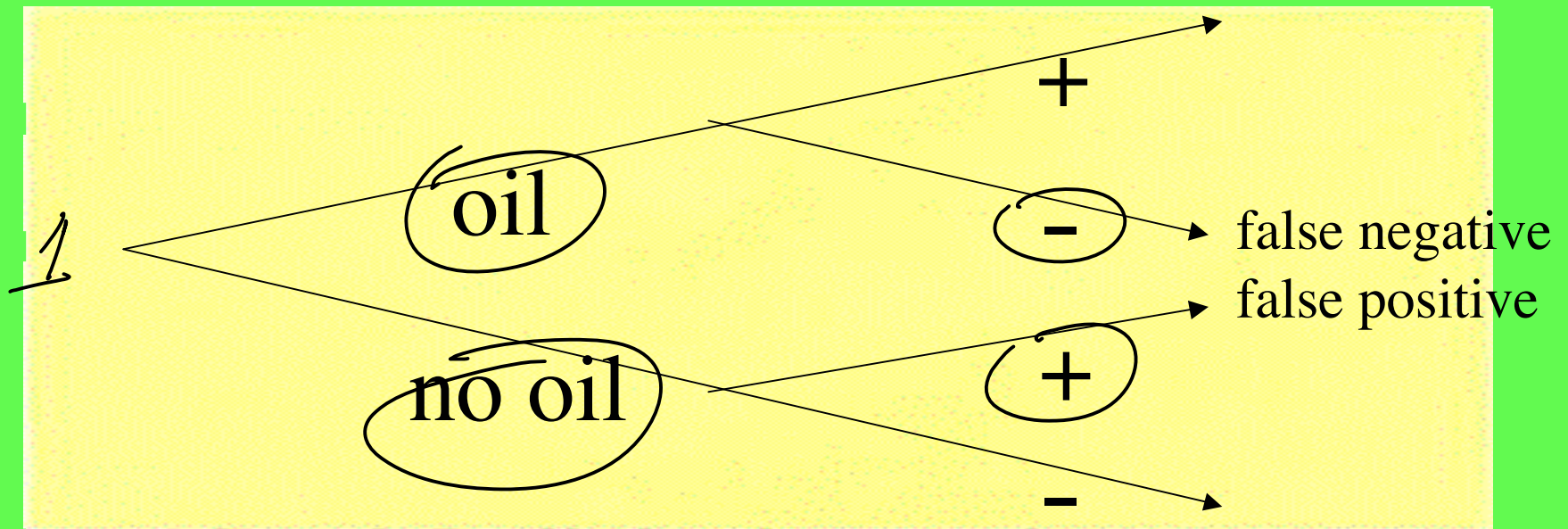
$P(B|A^c) = \frac{P(A^cB)}{P(A^c)}$
 $= .24 / .9$

TREE DIAGRAM

“oil” = oil is present

“+” = a test for oil is positive

“-” = a test for oil is negative



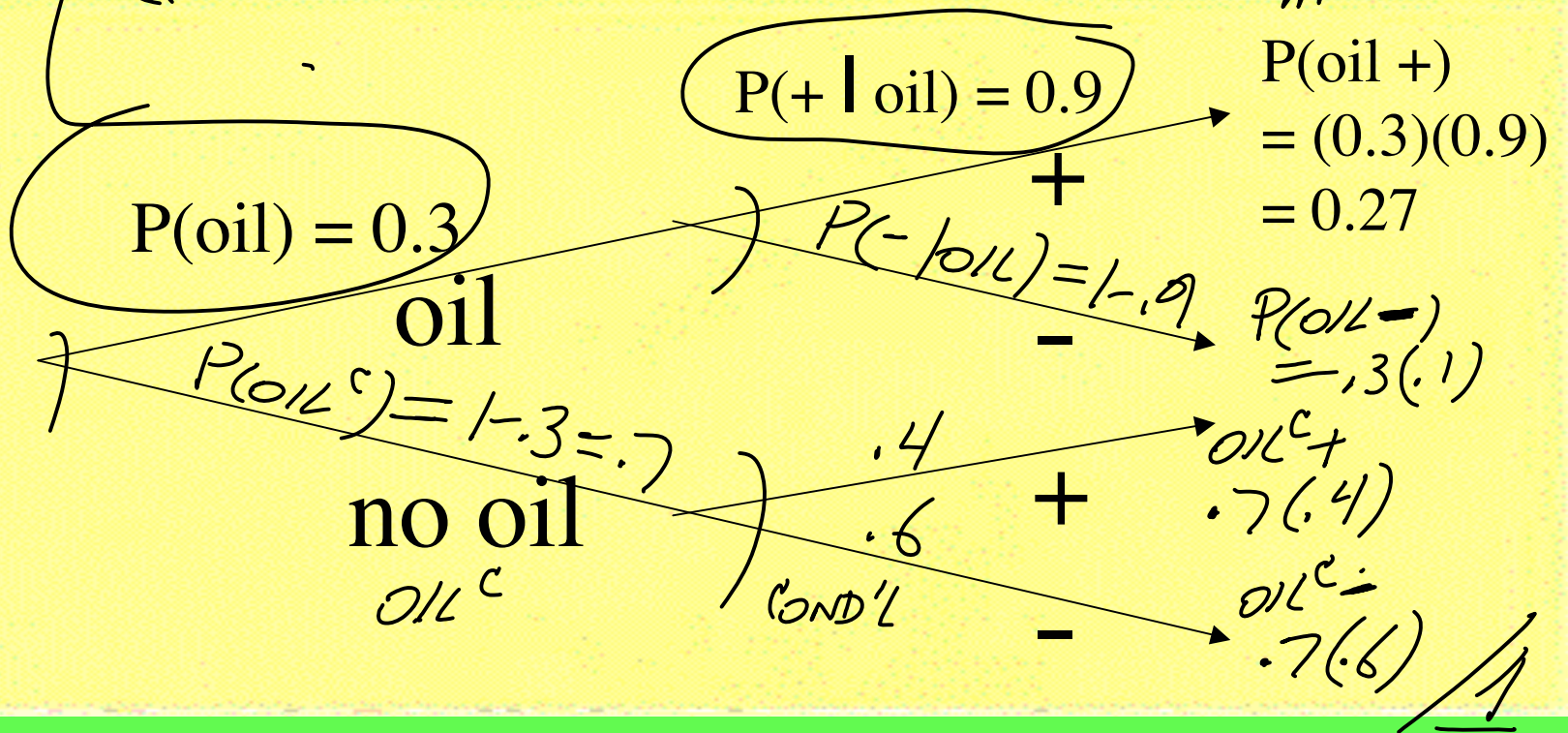
TREE DIAGRAM CONVENTIONS

$$P(\text{oil}) = 0.3$$

$$P(+ | \text{oil}) = 0.9$$

$$P(+ | \text{no oil}) = 0.4$$

MAULTREE

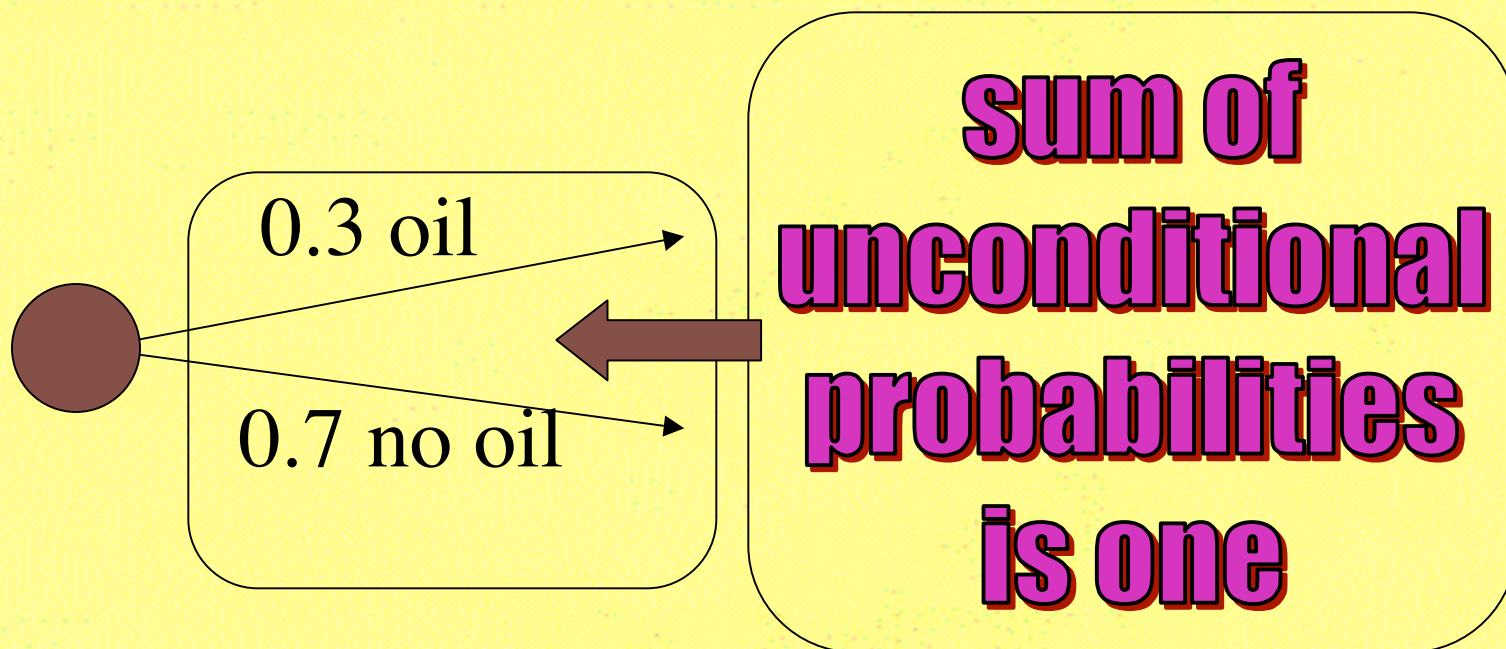


TOTAL OF BRANCHES = 1

$$P(\text{oil}) = 0.3$$

$$P(+ \mid \text{oil}) = 0.9$$

$$P(+ \mid \text{no oil}) = 0.4$$

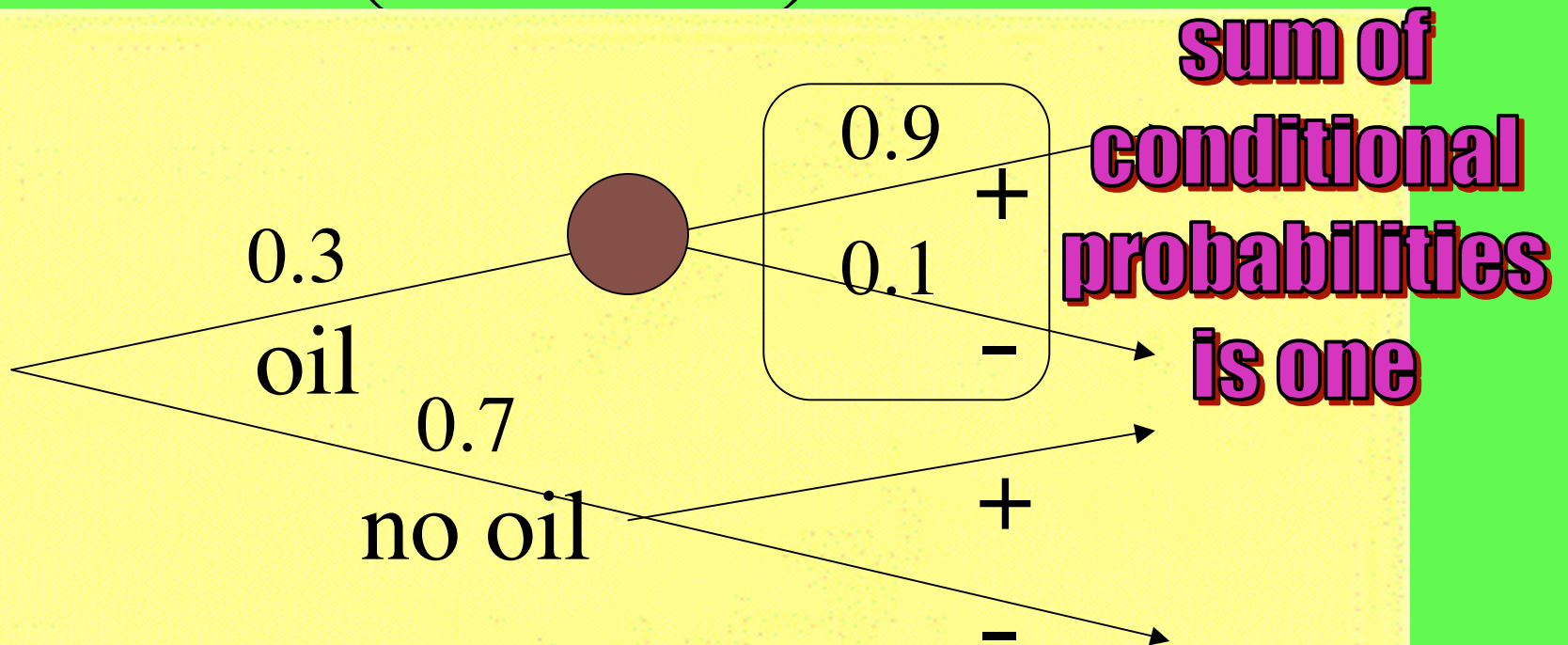


TOTAL OF CONDITIONAL BRANCHES = 1

$$P(\text{oil}) = 0.3$$

$$P(+ | \text{oil}) = 0.9 \quad P(- | \text{oil}) = 0.1$$

$$P(+ | \text{no oil}) = 0.4$$

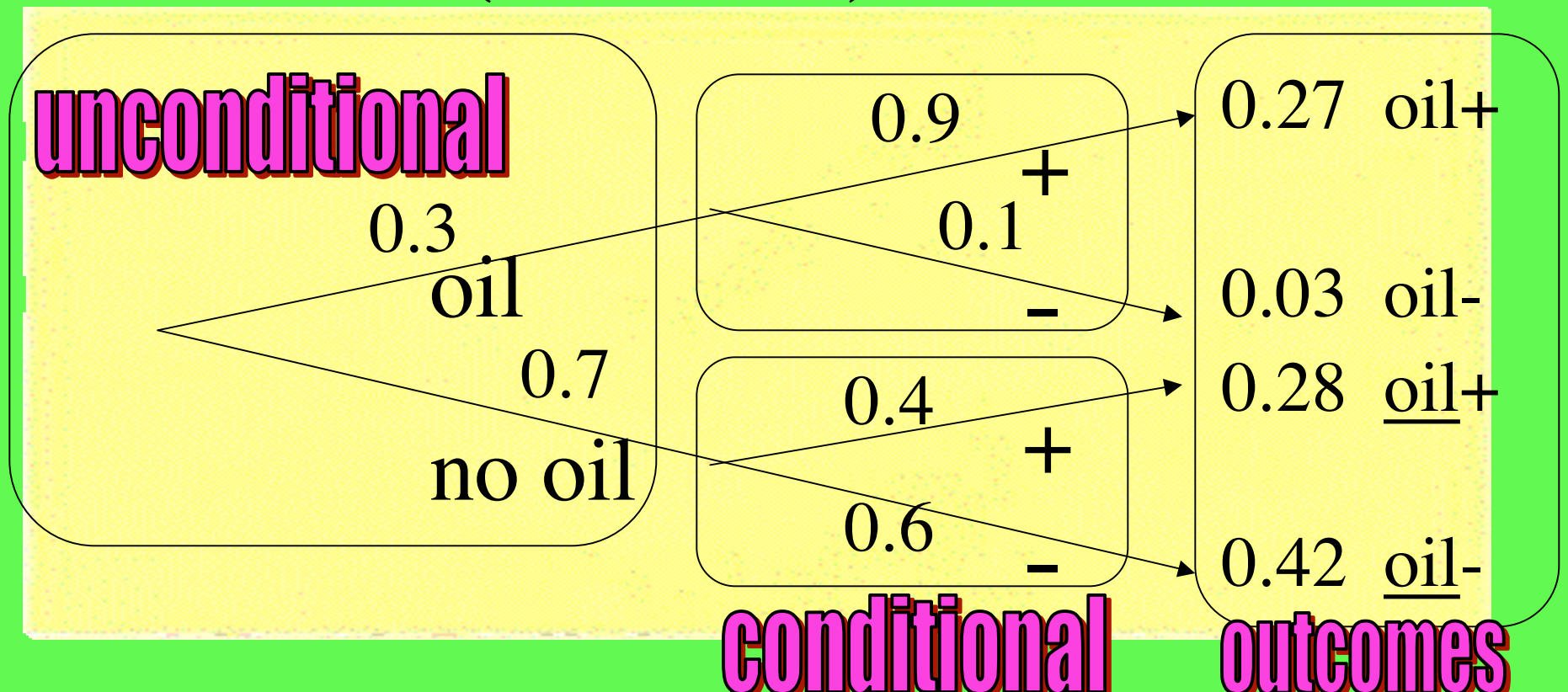


COMPLETE TREE

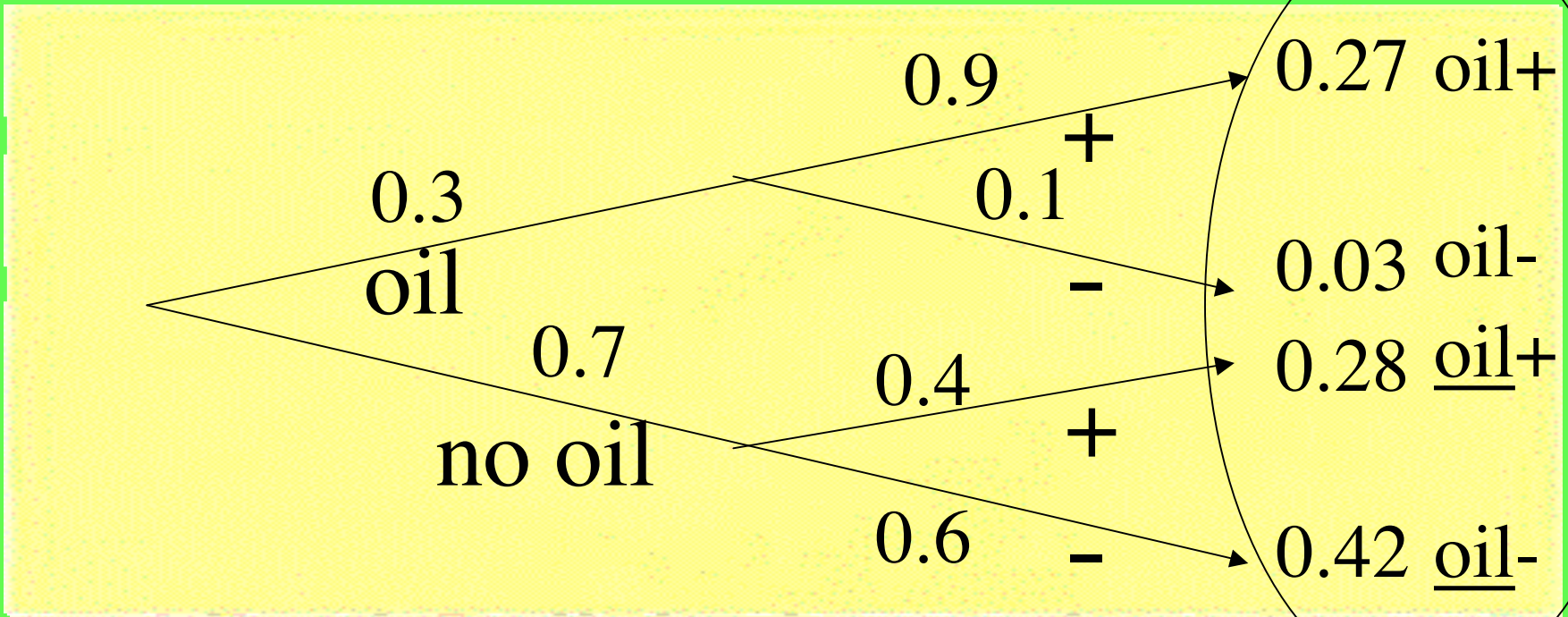
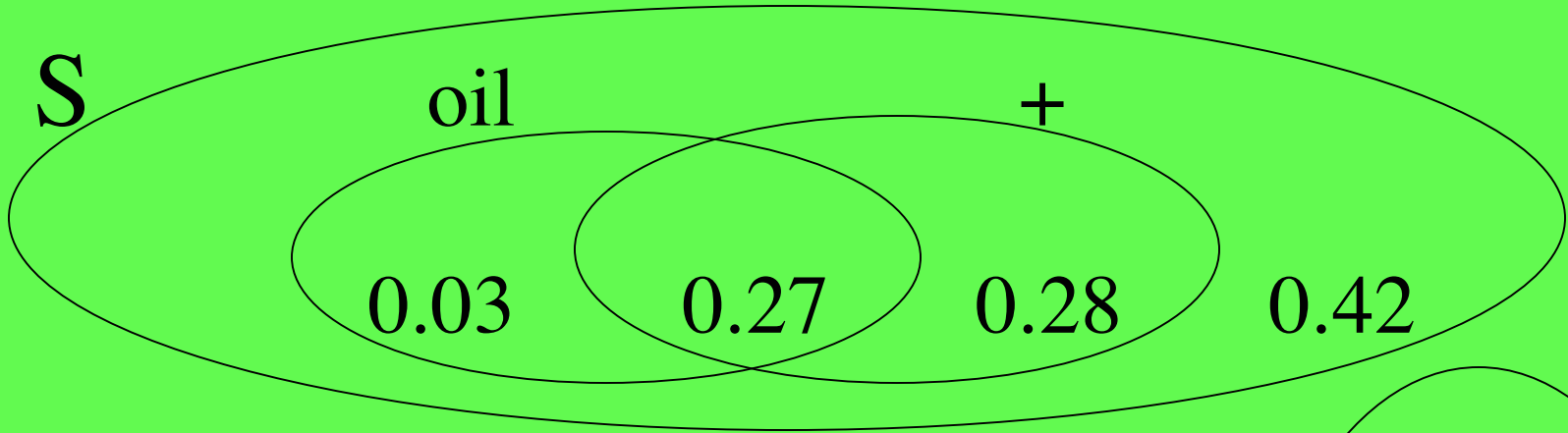
$$P(\text{oil}) = 0.3$$

$$P(+ \mid \text{oil}) = 0.9$$

$$P(+ \mid \text{no oil}) = 0.4$$



VENN DIAGRAM

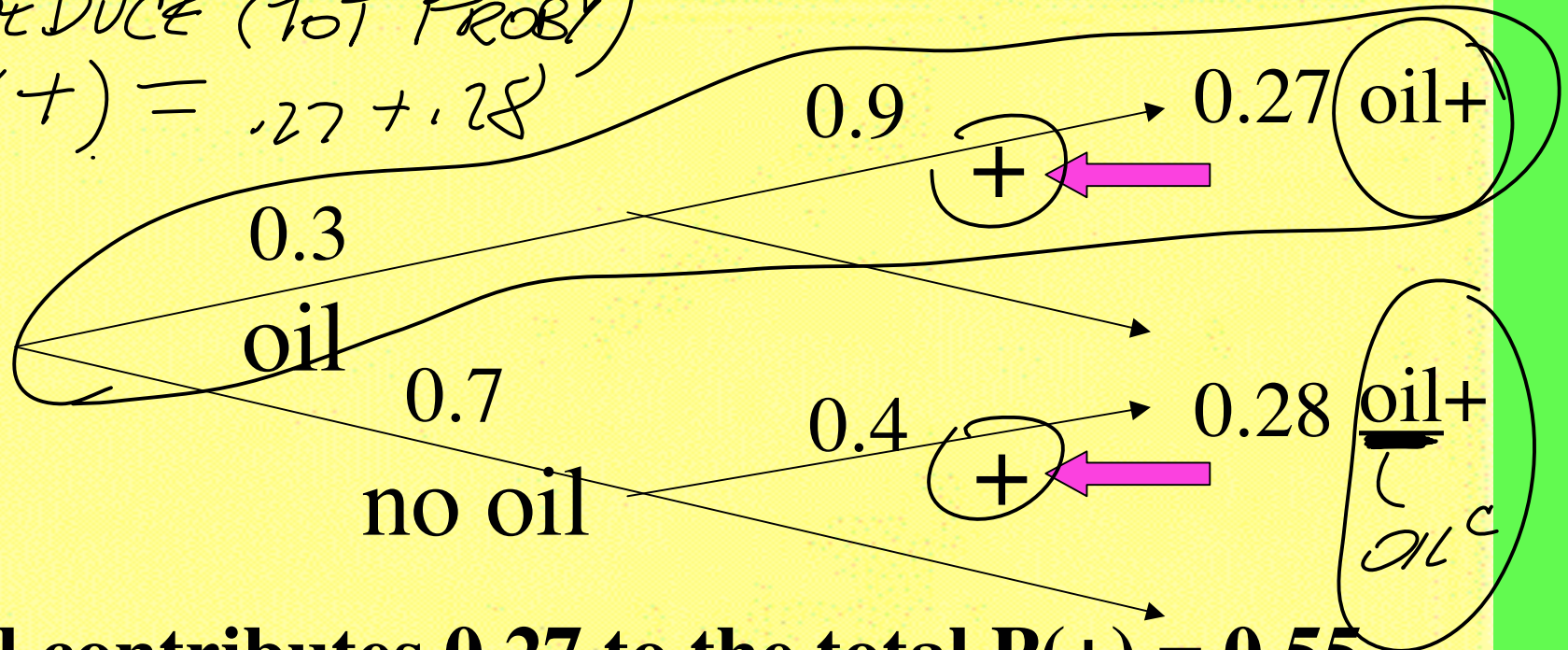


TOTAL PROBABILITY

$$P(+)=P(\text{oil}+) + P(\text{no oil}+)$$

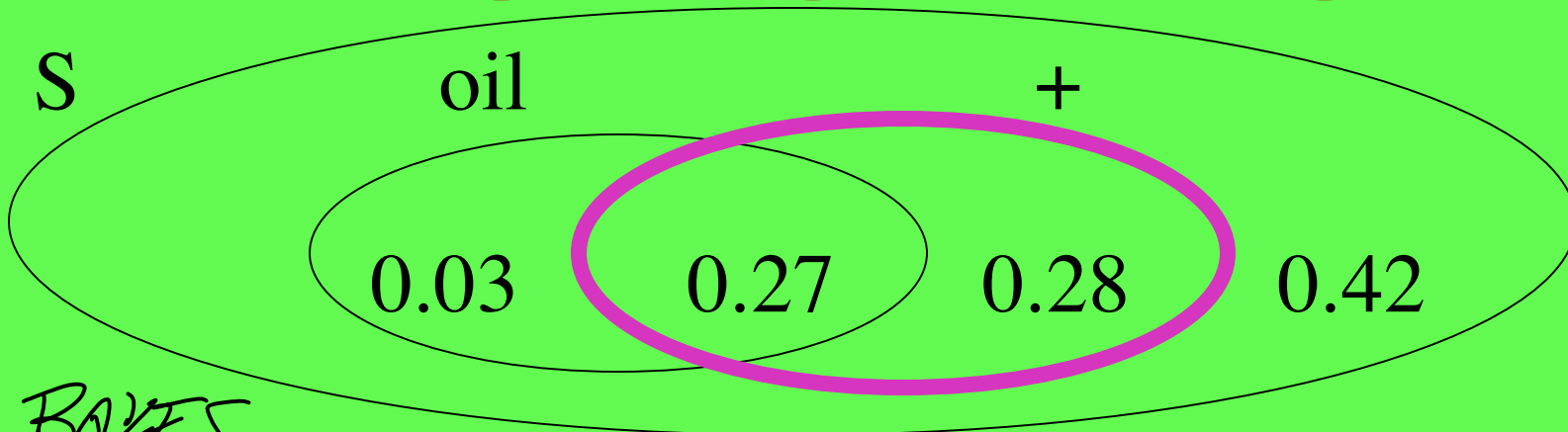
$$0.55 = 0.27 + 0.28$$

DEDUCE (TOT PROBY)
 $P(+)=.27+.28$



Oil contributes 0.27 to the total $P(+)=0.55$.

BAYES FORMULA



BAYES

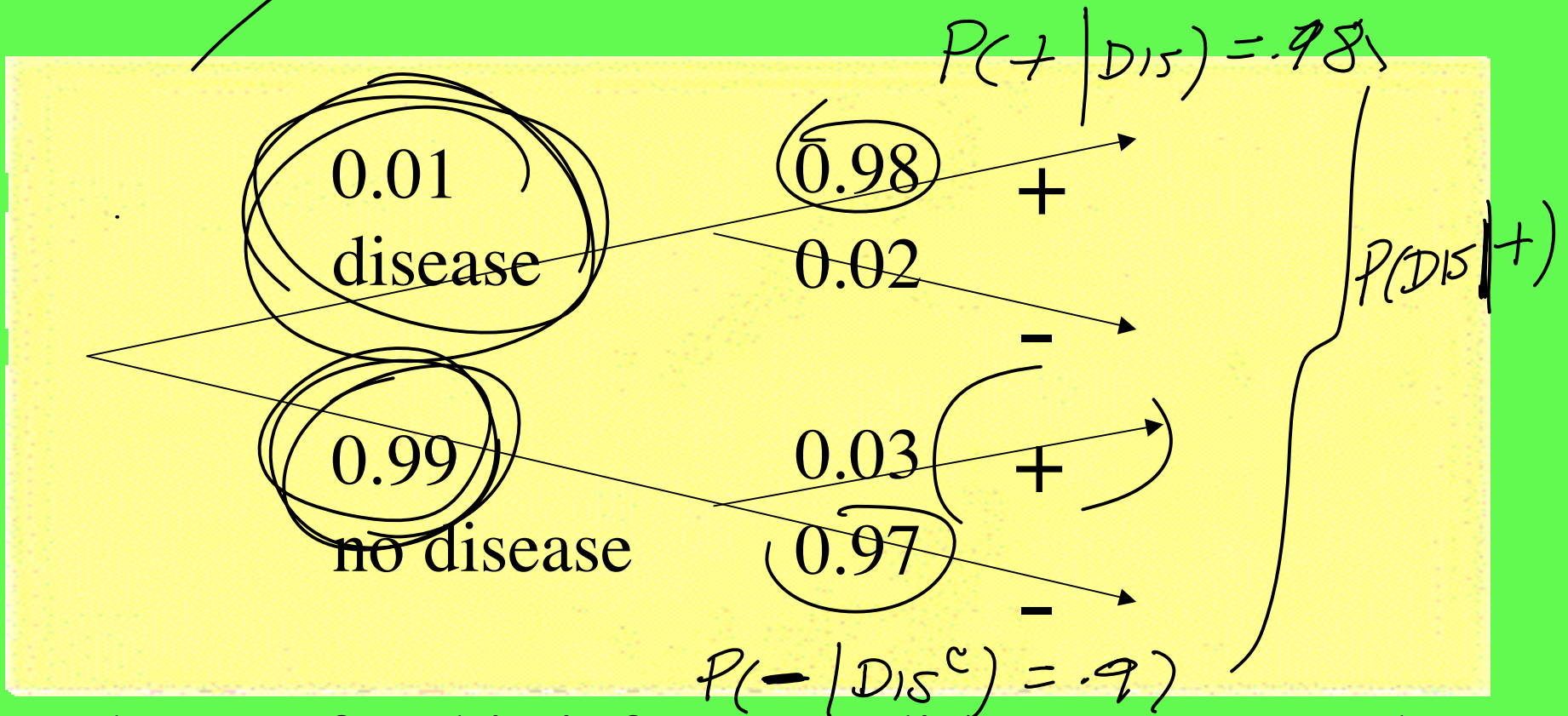
$$\begin{aligned} P(\text{oil} \mid +) &= P(\text{oil} \cap +) / P(+ \\ &= 0.27 / (0.27 + 0.28) \\ &= 0.4909.. \end{aligned}$$

0.27 oil+

0.28 oil+

Oil contributes 0.27 of the total $P(+)$ = 0.27+0.28.

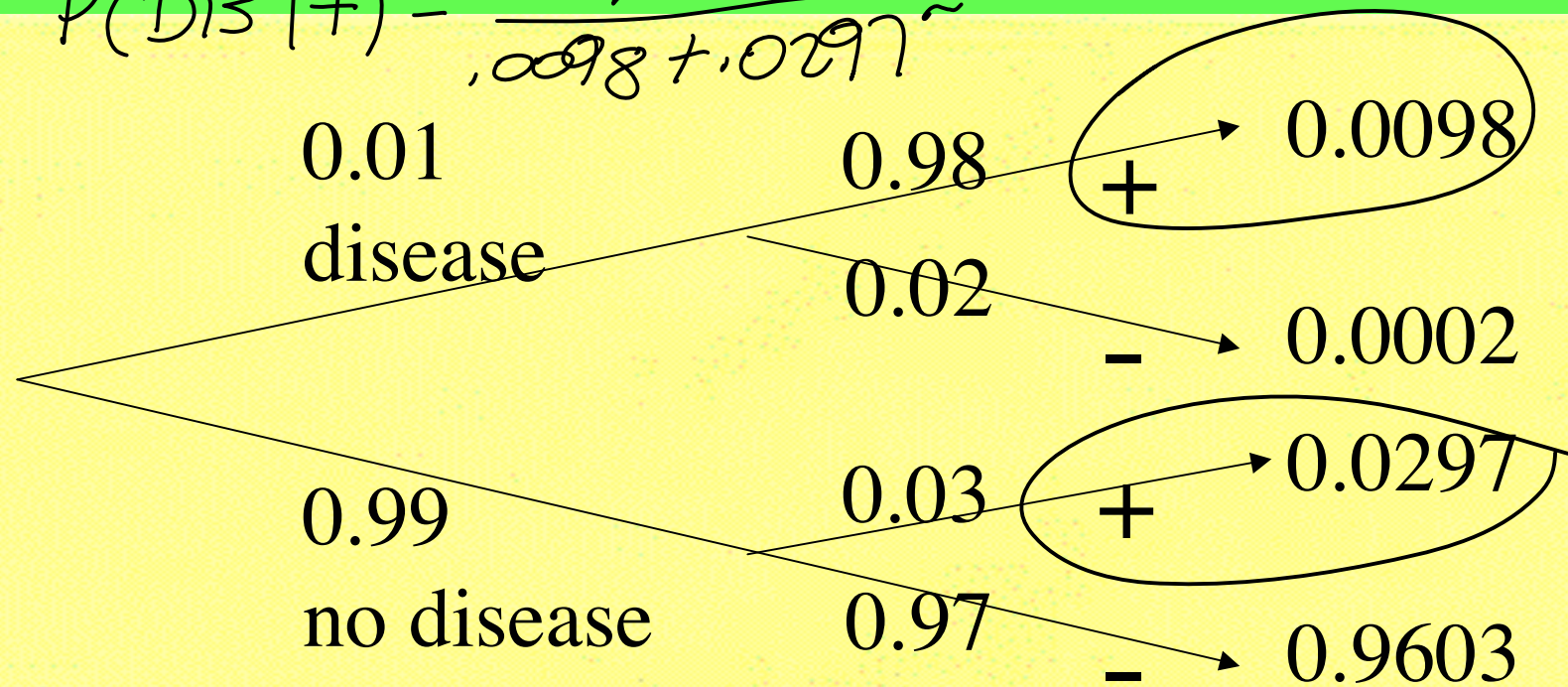
MEDICAL TEST



The test for this infrequent disease seems to be reliable having only 3% false positives and 2% false negatives. **What if we test positive?**

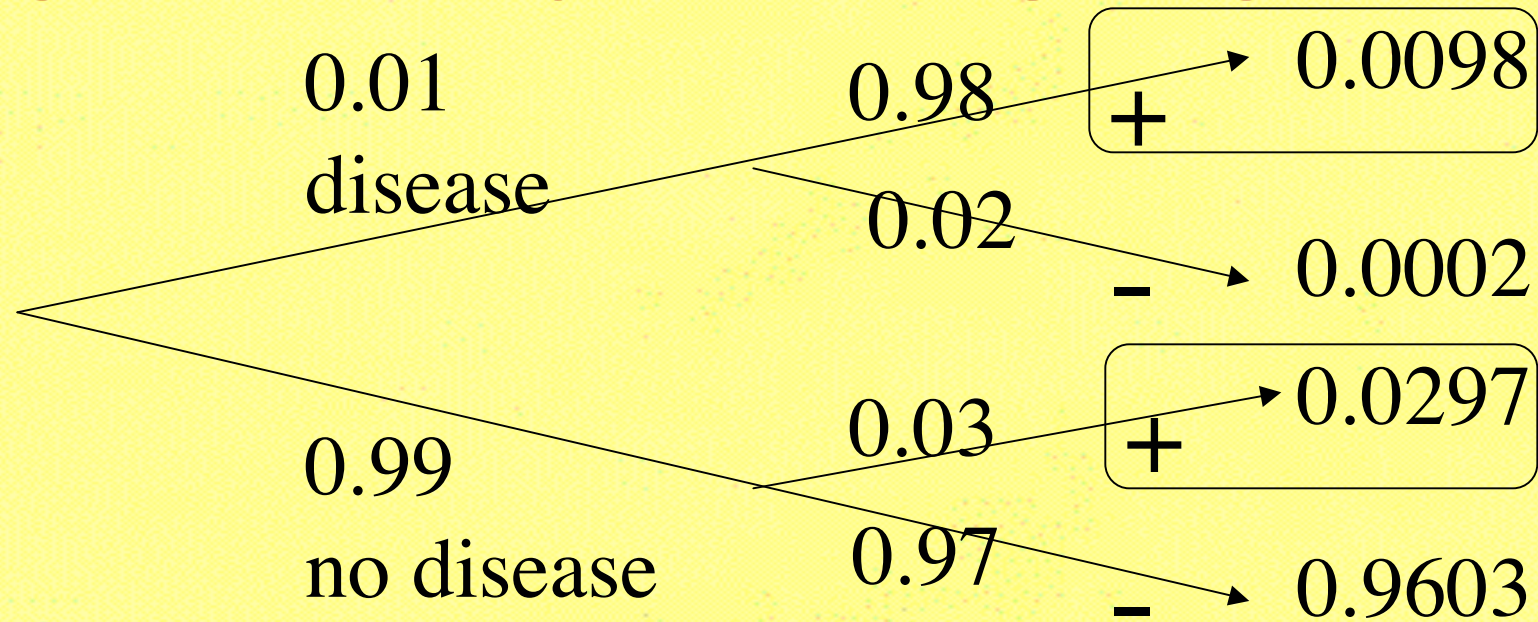
MEDICAL TEST

$$P(\text{Dis} | +) = \frac{0.0098}{0.0098 + 0.0297} \approx$$



We need to calculate $P(\text{diseased} | +)$, the **conditional probability** that we have this disease **GIVEN** we've tested positive for it.

CALCULATING OUR CHANCES OF HAVING THE DISEASE IF +



$$P(+) = 0.0098 + 0.0297 = 0.0395$$

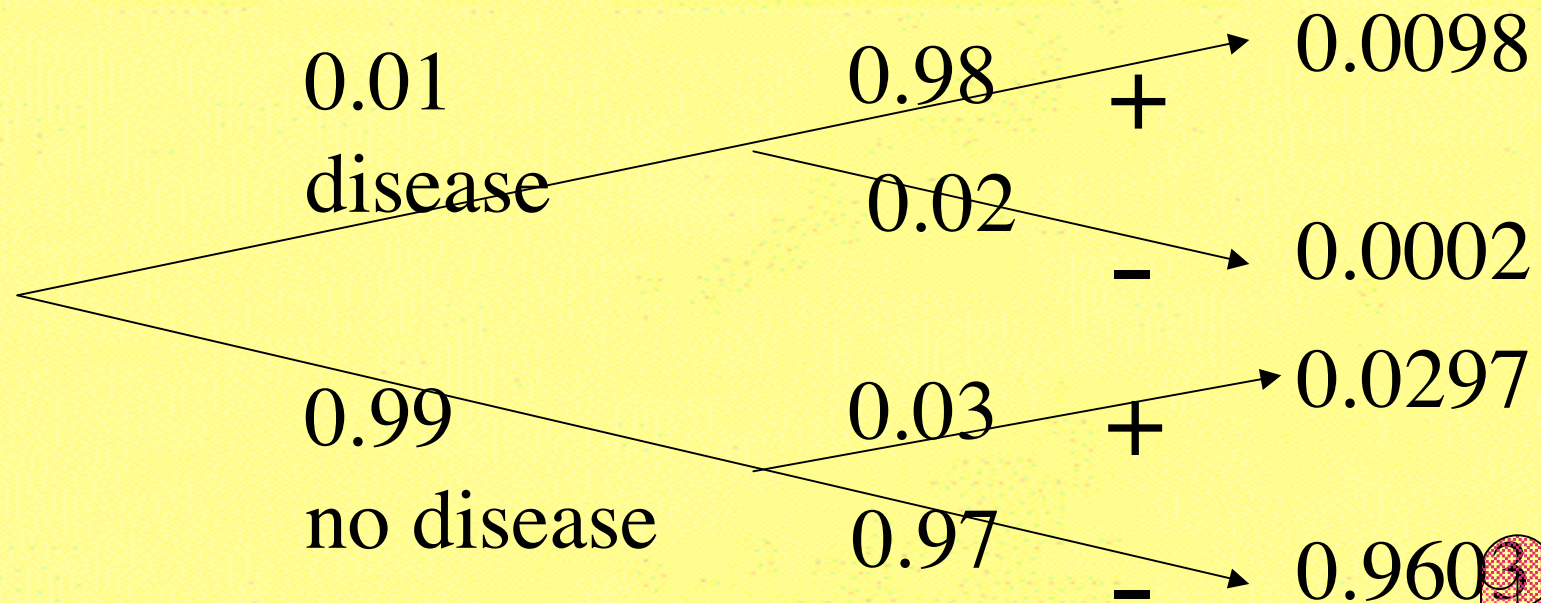
$$P(\text{disease} \mid +) = P(\text{disease}+) / P(+)$$

$$= .0098 / 0.0395 = 0.248.$$

only 25%!

FALSE POSITIVE PARADOX

one may overwhelm a good test by failing to screen

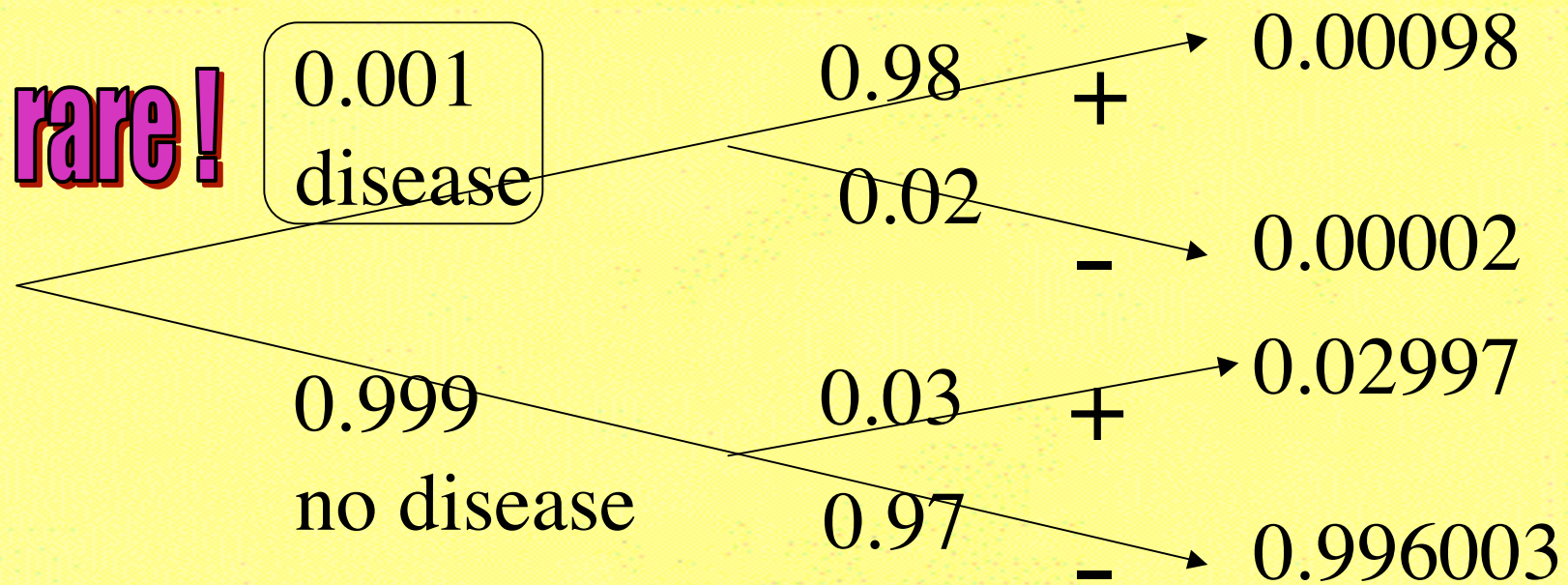


EVEN FOR THIS ACCURATE TEST:

$P(\text{diseased} \mid +)$ is only around 25% because the non-diseased group is so predominant that most positives come from it.

FALSE POSITIVE PARADOX

one may overwhelm a good test by failing to screen



WHEN THE DISEASE IS TRULY RARE:
P(diseased | +) is a mere **3.2%** because the huge non-diseased group has completely overwhelmed the test, which no longer has value

IMPLICATIONS OF THE PARADOX

FOR MEDICAL PRACTICE: Good diagnostic tests will be of little use if the system is overwhelmed by lots of healthy people taking the test. **Screen patients first.**

FOR BUSINESS: Good sales people capably focus their efforts on likely buyers, leading to increased sales. They can be rendered ineffective by feeding them too many false leads, as with massive **un-targeted sales promotions.**

